# Activity-selection Problem

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2019/06/11 @ TR-310-1, NTUST

# Review

- Dijkstra's algorithm maintains a set $S$ of vertices whose final shortest-path weights from the source $s$ have already been determined
  - $w(u, v) \geq 0$ for each edge $(u, v)$
    - Selects the vertex $u \in V - S$ with the minimum shortest path estimate
    - Adds $u$ to $S$
    - Relaxes all edges leaving $u$

- The Bellman-Ford algorithm returns a boolean value indicating whether or not there is a negative-weight cycle that is reachable from the source
  - If there is such a cycle, returns false
  - If there is no such cycle, the algorithm produces the shortest paths and their weights

# Introduction

- For many optimization problems, using dynamic programming to determine the best choices is overkill
  - More efficient algorithms will do

- A ***greedy algorithm*** always makes the choice that looks best at the moment
  - It makes a locally optimal choice in the hope that this choice will lead to a globally optimal solution
  - **Greedy algorithms do not always yield optimal solutions**, but for many problems they do

# Activity-selection Problem

- Suppose we have a set $S = \{a_1, a_2, \ldots, a_n\}$ of $n$ proposed ***activities*** that wish to use a resource
  - Each activity $a_i$ has a ***start time*** $s_i$ and a ***finish time*** $f_i$
  - Activities $a_i$ and $a_j$ are ***compatible*** if their execution times do not overlap
  - In the ***activity-selection problem***, we wish to select a maximum-size subset of mutually compatible activities
  - For example

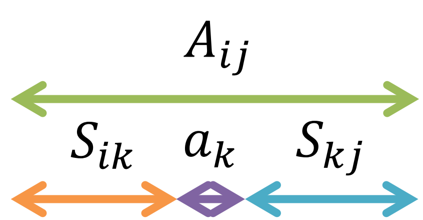
| $i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $s_i$ | 1 | 3 | 0 | 5 | 3 | 5 | 6 | 8 | 8 | 2 | 12 |
| $f_i$ | 4 | 5 | 6 | 7 | 9 | 9 | 10 | 11 | 12 | 14 | 16 |

    - The subset $\{a_3, a_9, a_{11}\}$ consists of mutually compatible activities
    - However, $\{a_1, a_4, a_8, a_{11}\}$ and $\{a_2, a_4, a_9, a_{11}\}$ are both largest subsets of mutually compatible activities

# DP for Activity-selection Problem.

- Let us denote by $S_{ij}$ the set of activities that start after activity $a_i$ finishes and that finish before activity $a_j$ starts
  - Suppose $A_{ij}$ is a maximum set of mutually compatible activities in $S_{ij}$


- If $A_{ij}$ includes activity $a_k$, the goal is left with two subproblems:
  - finding mutually compatible activities in the set $S_{ik}$
  - finding mutually compatible activities in the set $S_{kj}$

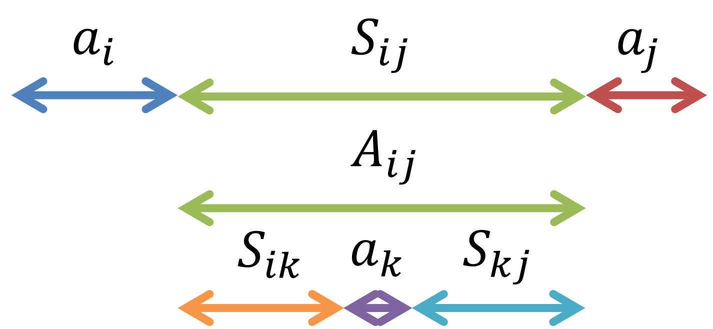


# DP for Activity-selection Problem..

- This way of characterizing optimal substructure suggests that we might solve the activity-selection problem by DP
  - If we denote the size of an optimal solution for the set $S_{ij}$ by $c[i, j]$, then we would have the recurrence

$$c[i,j] = c[i,k] + 1 + c[k,j]$$

$$c[i,j] = \begin{cases} 0, & if\ S_{ij} = \emptyset \\ \max\limits_{a_k \in S_{ij}} \{c[i,k] + 1 + c[k,j]\}, & if\ S_{ij} \neq \emptyset \end{cases}$$



# Greedy Search Strategy

- What do we mean by the greedy choice for the activity-selection problem?
  - Intuition suggests that we should **choose an activity that leaves the resource available for as many other activities as possible**
    - Of the activities we end up choosing, one of them must be the first one to finish
    - In other words, our intuition tells us to choose the activity in $S$ with **the earliest finish time**, since that would leave the resource available for as many of the activities that follow it as possible
  - If we make the greedy choice, we have only one remaining subproblem to solve: finding activities that start after $a_1$ finishes
    - We assume that the activities are sorted in monotonically increasing order of finish time

# Prove the Intuition

$A_k = \{a_j, \cdots\}$
$A'_k = \{a_m, \cdots\}$

- Let $S_k = \{a_i \in S: s_i \geq f_k\}$ be the set of activities that start after activity $a_k$ finishes. Consider any nonempty subproblem $S_k$, and let $a_m$ be an activity in $S_k$ with the earliest finish time. Then $a_m$ is included in some maximum-size subset of mutually compatible activities of $S_k$
  - Let $A_k$ be a maximum-size subset of mutually compatible activities in $S_k$
  - Let $a_j$ be the activity in $A_k$ with the earliest finish time
    - Remember that $a_m$ be an activity in $S_k$ with the earliest finish time
  - If $a_j = a_m$, we are done
  - If $a_j \neq a_m$, let $A'_k = A_k - \{a_j\} \cup \{a_m\}$
    - $A'_k$ is valid, because activities in $A_k$ are disjoint, $a_j$ is the first activity to finish in $A_k$, and $f_m \leq f_j$
    - Thus, $|A'_k| = |A_k|$, we conclude that $A'_k$ is a maximum-size subset of mutually compatible activities of $S_k$, and it includes $a_m$

# Questions?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